Syllabus of PhD Preliminary Examination in Numerical Analysis[1]

NA Prelim Committee
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The PhD Preliminary Examination in Numerical Analysis is intended to determine whether a student has adequate knowledge at the undergraduate level in numerical analysis including numerical linear algebra to begin a Ph.D. thesis research in applied mathematics. The exam is written and graded by a committee of professors in the Department of Mathematics with expertise in numerical analysis.

The written exam consists of approximately 6 to 8 questions. The students are given four hours to take the exam. Students are not allowed use of notes or books, but allowed use of a calculator. A passing score is 70% or higher. Students who fail the exam can take the exam one more time.

Students interested in taking the exam should have taken the courses, Math 410 − Numerical Methods and Math 411 − Numerical Linear Algebra, or their equivalents. A student should prepare for the exam by studying the relevant material from several books listed at the end of this syllabus, and by reviewing problems of the practice test and of previously given exams.

Numerical Analysis

1. **General concepts**

   Floating point numbers and operations, roundoff errors, loss of significance, degree of precision of an approximation formula, Richarsdon extrapolation, the Aitken acceleration, Taylor series, Chebyshev polynomials, convergence of sequences, convergence orders, computational complexity of algorithms.

2. **Interpolation**

   Lagrange interpolation polynomial, divided differences, Chebyshev points, Hermite interpolation, spline interpolation, error analysis, algorithms and costs.

3. **Function and data approximation**

   Continuous and discrete least squares approximations, orthogonal polynomials, Gram-Schmidt process, trigonometric least squares approximation, trigonometric interpolation, discrete Fourier transforms, fast Fourier transform algorithm.

4. **Numerical differentiation**

   Finite difference formulas, differentiation using polynomial interpolation, error analysis and sensitivity to roundoff errors.

5. **Numerical integration**

   Closed and open Newton-Cotes quadratures, Gaussian quadrature, composite formulas, Romberg integration, adaptive integration, error analysis, algorithms, and costs.

6. **Methods for finding zeros of functions of one variable**

   Bisection method, fixed point iteration, Newton's method, secant method, convergence analysis of iterative methods.

7. **Methods for finding zeros of functions of several variables**

   Fixed point iteration, Newton's method, steepest descent method, homotopy and continuation methods.

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8. **Methods for solving initial value problems for systems of ODEs**

   Euler's method, Taylor method, Runge-Kutta methods, Runge-Kutta-Fehlberg method, multistep methods, predictor-corrector methods, stability of one step and multistep methods, stiff equations, A-stability.

9. **Finite difference method for Poisson's equation**

   Five-point finite difference approximation of the Laplace operator, approximation of the Dirichlet problem.

Numerical Linear Algebra

1. **General concepts**

   Vector spaces, vector and matrix algebra, basic linear algebra algorithms, vector and matrix norms, condition numbers, well and ill posed problems.

2. **Triangular systems**

   Methods for solving lower and upper tringular systems.

3. **Cholesky factorization**

   Positive definite matrices, Cholesky factorization, band Cholesky, costs.

4. **Gaussian elimination**

   Gaussian elimination, pivoting, LU factorization and its variants, band matrices, costs.

5. **QR factorization**

   Orthogonal matrices, plane rotators and reflectors, QR factorization,

6. **Gramm-Schmit process**

   Gramm-Schmit process and its relation to QR factorization.

7. **Least squares problem**

   Solving least squares problems, full rank and rank deficient cases.

8. **SVD**

   SVD decomposition, properties of singular values, pseudoinverse, SVD and least squares problem, minimal solutions.

9. **Eigenvalues and eigenvectors**

   Power method and its variants, shifting, Rayleigh quotient iteration, similarity transformations, unitary, Hermitian, and normal matrices, spectral theorems, Schur's theorem, reduction to Hessenberg form and tridiagonal forms, Francis's algorithm, computation of eigenvectors

10. **Iterative methods for linear systems**

    Classsical interative methods – Jacobi, Gauss-Seidel, SOR, SSOR, convergence analysis, Krylov subspaces, Arnoldi and Lanscos algorithms, steepest descent and conjugate gradient methods, preconditioning.

References

Main

1. R. Burden and J. Faires, Numerical Analysis, Brooks/Cole, 9 edition, 2010.
2. W. Cheney and D. Kincaid, Numerical Mathematics and Computing, Brooks/Cole, 7th edition, 2012.
3. D. Watkins, Fundamentals of Matrix Computations, John Wiley & Sons, 3rd edition, 2010.

Supplementary

1. U. Ascher and C. Greif, A First Course in Numerical Methods, SIAM, 2011.
2. G. Dahlquist and A. Bjork, Numerical Methods, Dover, 2003.
3. J. Demmel, Applied Numerical Linear Algebra, SIAM, 1997.
4. G. Golub and C. Van Loan, Matrix Computations, 4th edition, Johns Hopkins University Press, 2012.
5. D. Kincaid and W. Cheney, Numerical Analsyis: Mathematics of Scientific Computing, 3rd edition, AMS, 2002.
6. L. Trefethen and D. Bau, Numerical Linear Algebra, SIAM, 1997.